Oscillations

1. A coin rests on the top of a piston that executes simple harmonic motion vertically with an amplitude of 10 cm. At what minimum frequency does the coin lose contact with the piston?

2. A small particle slides inside a frictionless spherical bowl of radius R, as shown in Fig.1. (a) Show that the motion is simple harmonic for small displacements from the lowest point. (b) What is the period?

3. Water fills a length l of a U tube, as in Fig.2. The water is slightly displaced and then allowed to move freely. (a) Show that the liquid executes simple harmonic motion. (b) What is the period?

4. Fig.3. shows a tunnel in a uniform planet of mass M and radius R. At a distance r from the center, the gravitational attraction is due only to the sphere of radius r. Thus,

$$F = \frac{GmM(r)}{r^2} = \frac{mgr}{R}$$

where $M(r) = Mr^3/R^3$ and $g = GM/R^2$. Show that Newton's second law for the motion along the tunnel leads to the differential equation for simple harmonic motion $\frac{d^2x}{dt^2} + \frac{g}{R}x = 0$. Estimate the period of the oscillation for the earth.

5. A mass m is connected to two springs of force constants k_1 and k_2 as in Fig.4 (a) and (b).In each case, the mass moves on a frictionless table and is displaced from equilibrium and released. Show that in each case the mass exhibits simple harmonic motion with periods

$$(a) \quad T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}} \qquad (b) \quad T = 2\pi\sqrt{\frac{m}{k_1+k_2}}$$

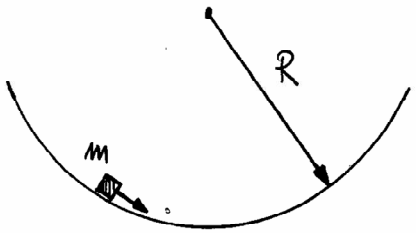

Fig.1.

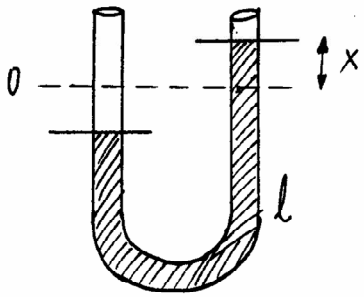

Fig.2.

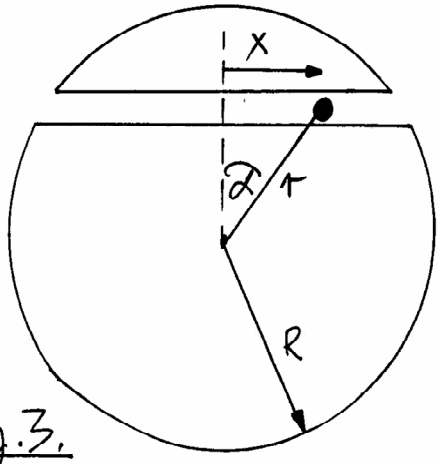

Fig.3.

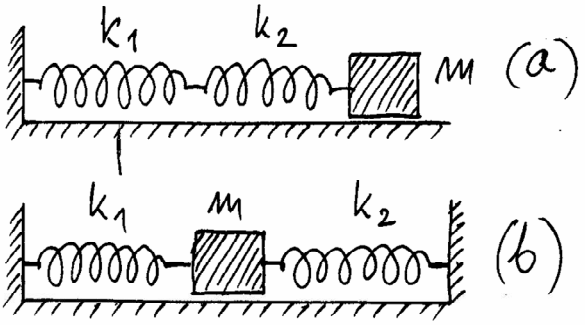

Fig.4

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